Accelerating Convergence of Unstable Resonator Calculations with a Gain Medium

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Introduction

A SIMPLE technique for improving the convergence rate of the calculation of unstable resonators with a gain medium is presented. The technique was found to be very useful in reducing the required computer time in our study where the population inversion was created by chemical reactions occurring in the flowing medium. The technique can also be applied to other types of gain medium. The larger the ratio of the time required for making a gain calculation to that for making a propagation calculation within the unstable resonator is, the more useful the technique will be. In our study,[1] this ratio is typically of the order of 10^2.

Calculations for unstable resonators with very complex gain mediums are exceedingly difficult to make because the flow and radiation fields are coupled together. The problem is usually made tractable by assuming that the amplification and propagation processes can be separated into two parts or modules and then iterating back and forth between the two modules until the results converge. In the amplification (GAIN) module, by using a specified distribution of the incident radiation fields at the edge of the gain medium, the flowfield and the gain distribution are computed and the amplified radiation fields are determined in the process. In the propagation (PROP) module, the now-amplified beam is propagated from the edge of the gain medium, across the resonator, and back again to the edge of the gain medium. The procedure is repeated until the flow and the radiation fields converge sufficiently. In computer codes where the flowfield is modeled in detail, the computer time for one GAIN execution could be 100 times longer than that for one PROP execution. It is therefore highly desirable to devise a technique that would reduce the total number of GAIN calculations required in obtaining a converged solution. The most obvious technique of reusing the gain distribution from a previous iteration does not take into account the decrease in gain as the intensity increases. This Note presents a simple technique that better accounts for this gain-intensity interplay.

Analysis and Results

In this study,[1] the amplification (GAIN) module was the LAMP code[2] modified only to use a known intensity distribution. The propagation (PROP) module was a two-dimensional (cylindrical) unstable resonator code,[1] which made use of the Fast Fourier Transform (FFT) propagation procedures described by Salvi[3] and Phelps.[4] However, the technique is quite general and not restricted to these particular modules.

We assume first that the amplification and propagation modules are distinct entities linked by the radiation fields passed back and forth between them during the iterative process. The effect of the radiation fields on the flow appears explicitly in the energy and the species equations only in terms of the product of the gain and the total radiation intensity, gI, for each transition (see, for example, Ref. 2). Therefore, if the distribution of gI is fixed from one iteration to the next, the flow will also be fixed or "frozen." We also make use of the fact that, even for the simplest case of an empty (no-gain-medium) unstable resonator, many iterations are required before the radiation fields converge. We therefore pass the radiation fields back and forth several times between the propagation module and an alternate amplification module appropriate to a "frozen" flow, which allows the radiation fields to converge somewhat before the flowfield is changed. This procedure tends to increase the total number of amplification-propagation iterations. However, if the alternate amplification module used for some of these iterations is significantly faster than the GAIN module, an appreciable saving of computer time can be obtained. Noting that the propagation module takes the output beam at the edge of the gain medium and returns the beam incident upon the edge of the gain medium, a suitable alternate amplification module (CONGI) can be constructed by requiring that, at every point along the edge of the gain medium, the product gI is equal to its value from a previous iteration, i.e.,

$$gI = (gI)_{\text{previous}} \tag{1}$$

where g is the average gain across the medium of thickness Δ and I is the total intensity at the edge of the gain medium,

$$I = I_{\text{in}} + I_{\text{out}} \tag{2}$$

The incident and the outward (amplified) intensities are related by

$$I_{\text{out}} = I_{\text{in}} e^{2g\Delta} \tag{3}$$

where, for simplicity, the concave mirror has been assumed to be perfectly reflecting. For given incident radiation fields, the GAIN module can be used to calculate the amplified radiation fields and determine the RHS of Eq. (1) for each transition. These amplified lines can then be propagated by the PROP module to yield improved values for the incident radiation fields, F_{in}, and intensity, I_{in}. The alternate amplification module (CONGI) can now be used to calculate I, I_{out}, and g from Eqs. (1-3), and the amplified radiation fields, F_{out} $[=F_{\text{in}} \exp(g\Delta)]$. The amplified fields can now be propagated by again using the PROP module. Using the Newton-Raphson method, Eqs. (1-3) are solved very rapidly and the CONGI

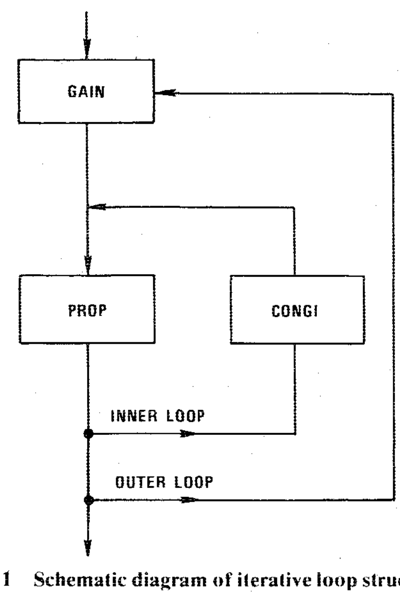

Fig. 1 Schematic diagram of iterative loop structure.

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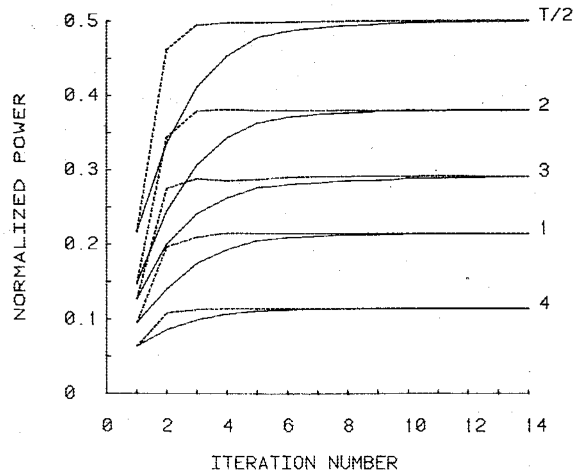

Fig. 2 Convergence history of output power in each vibrational band and their total normalized by the total output power for the 14th iteration using CONGI technique.--with CONGI,—without.

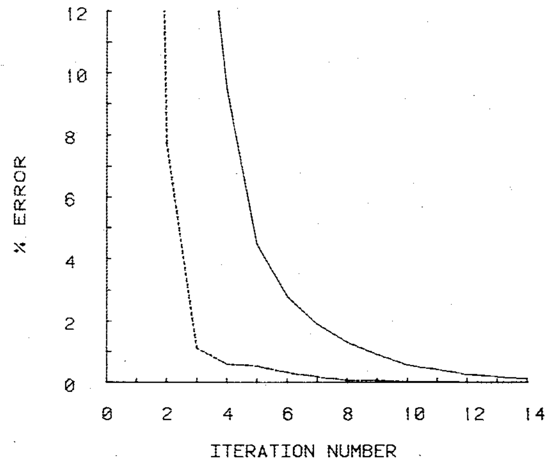

Fig. 3 History of percent error (deviation from the 14th iteration) of total output power.--with CONGI,—without.

module takes even less computer time than the PROP module and only about 1/100th of that of the GAIN module.

As shown schematically in Fig. 1, an inner loop is formed in which the CONGI-PROP cycle can be repeated any number of times before the propagated radiation fields are sent back to the GAIN-PROP (outer) loop. If the inner loop is repeated enough times, the radiation fields will converge. However, unless the flowfield has already converged, it is not very meaningful to continue the iteration of radiation fields in the inner loop until convergence. We have found that the radiation fields change relatively little after the first few iterations of the inner loop. Consequently, we have used the procedure of repeating the inner loop four times for every iteration of the outer loop. Since the CONGI module runs so much faster than the GAIN module, we have not attempted to determine the optimal number of inner-loop iterations. Actually, the optimum may very well depend on the particular case under study.

To demonstrate the effectiveness of the present technique in improving the convergence rate, the base case of Ref. 1 is recomputed with and without the CONGI-PROP inner loop. The starting radiation fields are uniform in the flow direction and the same for each of the 24 lasing transitions (six transitions for each of four vibrational bands). The convergence history of the total output power and the output power of the four individual bands, each normalized by the final value of the total output power, is shown in Fig. 2. Results obtained without using the CONGI-PROP inner loop are shown by solid lines and those with the inner loop are shown by dashed lines. The number of iterations shown corresponds to the number of the outer loops. The same starting conditions have been used for the two cases. The first iteration results of the two methods are identical because a GAIN-PROP loop is needed to determine $(gI)_{\text{previous}}$ for the CONGI-PROP loops. Figure 2 shows the effectiveness of the present technique in reducing the total number of iterations required. However, this is indicated more clearly in Fig. 3, which shows the percent error defined as the deviation of the total output power from that of the 14th iteration using the CONGI technique. For an accuracy of approximately 1%, only three iterations are needed using the CONGI technique. This saves about six iterations and two-thirds of the computer time required otherwise. Since one iteration takes approximately 15 min on a CDC 6500 computer, the cost saving obtained by using the CONGI technique is very substantial.

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Ignition of a Combustible Mixture by a Hot Particle

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Introduction

THE ignition of a combustible mixture by either hot or incendiary particles is a major cause of fires and explosions in mines, machine shops, and also in fuel tanks within both civilian and military vehicles. Whereas some experiments[1,2] have been conducted aiming to characterize the flammability of the mixture as a function of the ignition source, a reasonably rigorous theoretical analysis has not been attempted but is needed. We do so herein for the case when the ignition source is a hot, inert, particle; using matched asymptotic analysis in the realistic limit of large activation energies. It will be demonstrated that, through an appropriate transformation, the governing equations for the present problem can be cast to essentially the same forms as those for a similar problem[3] recently analyzed for the stagnation point ignition of a premixed combustible, such that existing solutions can be readily utilized. In particular, an explicit ignition criterion is derived.

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